T-matrix simulations of Spectral Polarimetric Variables from a cloud-radar

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1 Introduction

Cloud radar observations and spectral polarimetry are crucial for understanding cloud microphysics. The overall purpose of this study is two-fold: (a) to describe the methodology for simulating polarimetric spectral variables including white and stochastic noise of a real radar spectrum, as well as the impact of atmospheric turbulence and (b) to compare simulations with observed spectra for rain observations. Rain electromagnetic scattering properties have been historically computed by assuming spheroidal shapes via the T-matrix method (Mishchenko et al., 2000). Such models have been found satisfactory to explain radar and radiometeric measurements. However, raindrops generally change due to oscillations, which cause departure from rotationally symmetric shape, and make T-matrix tools impractical since they hinge upon the assumption of rotationally symmetric particles.

This work focuses on generating simulations of a 94 GHz cloud radar observations in rain conditions, pointing at 45 degrees and comparing with real observations. The spectral differential reflectivity (sZ_{DR}) and spectral differential phase ($s\delta_{hv}$) are the variables of interest. They are produced with the T-matrix method, by computing the electromagnetic scattering properties and simulating the radar response.

The simulation tool is described in section 2 and explores diverse conditions, allowing for the modification of rain rate, white and spectral noise, and turbulence parameters. The effect of atmospheric turbulence introduces an increased spread of velocities within the radar volume and contributes to the blurring of the spectral features, such as smearing out the distinct features (Mie scattering notches) in the Doppler spectrum. Incorporating the impact of turbulence in the simulations for spectral polarimetric variables is a complex task and the attempt is discussed in this study.

2 Methodology

The simulations are generated by using a Python package for computing the electromagnetic scattering properties of nonspherical particles using the T-matrix method (Leinonen, 2014), exclusively targeting in rain conditions. The backscattering amplitude matrix, S, and the phase matrix, Z, are calculated for drops of different diameters D, with axis ratios parameterized as following: Very small droplets are conceived as perfect spheres (axis ratio = 1), and as their size increases, they are modelled as spheroidal particles and an oblate shape is assumed (axis ratio > 1). The scattering geometry of the simulation corresponds to a radar pointing at a 45-degree elevation angle. Raindrops are assumed to be partially aligned with their maximum dimension preferentially on the horizontal plane: scattering properties are averaged over Gaussian distributions of canting angles with different standard deviations.

Firstly, the single-particle Polarimetric Variables are computed: the backscattering cross sections for Vertically and Horizontally polarized radiation (σ_{VV}, σ_{HH}), the differential scattering phase δ_{hv} and the co-polar correlation coefficient ρ_{hv}.

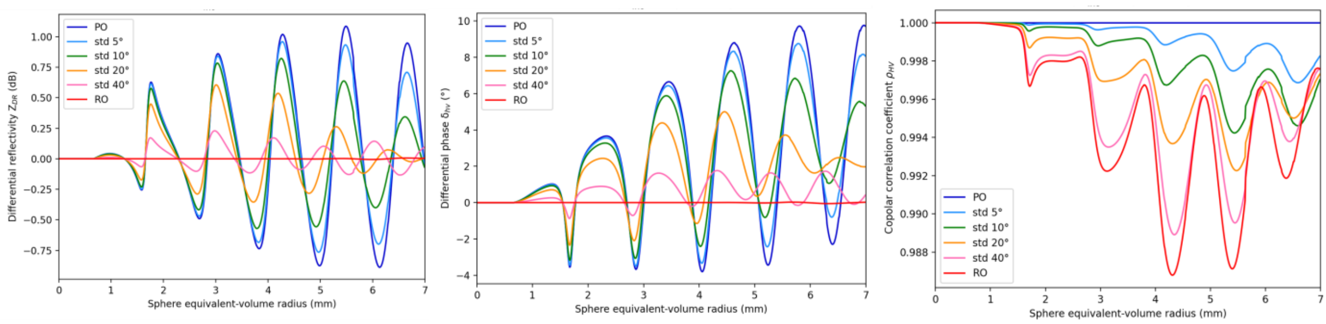

***Figure 1**: Left: Differential reflectivity Z_{DR}, Middle: Differential phase δ_{hv}, Right: Copolar correlation coefficient ρ_{hv} as a function of sphere equivalent-volume diameters, for a 94 GHz radar pointing at 45◦. For the T-matrix method, a complex refractive index of water at 10◦C temperature is assumed. PO, RO stand for Perfect Orientation and Random Orientation, respectively.*

Then, an ideal spectrum S_{VV} (1) for the V-channel is independently generated for each diameter D. A Gamma distribution is used to represent the variability of a natural rainfall Drop Size Distribution N(D).

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$$S_{VV}(D) = \frac{\lambda^4}{\pi^5 |K|^2} N(D)\sigma_{vv}(D)\frac{dD}{du} \quad (1)$$

λ is the radar wavelength, $|K^2|$ is derived from the dielectric factor of water, N(D) is the drop size distribution, σ_{VV} is the backscattering cross section for V channel and u is the velocity of droplets along the line of sight of the radar beam. Following Yu et al. (2012), the complex voltage signal in the V channel in the velocity domain can be written as:

$$V_V(u) = \sqrt{S_{VV}(u)\ln g^{[1]}}e^{i\theta^{[1]}} \quad (2)$$

where $g^{[1]}$ and $\theta^{[1]}$ are independent, identically distributed random variables with uniform distribution between 0 and 1 and between $-\pi$ and π, respectively. The time series of complex voltage signal can be obtained via an inverse FFT of V_V. This process can be repeated iteratively to generate independent stochastic realizations of the same spectrum. Similarly, for the H channel in the velocity domain:

$$V_H(u) = \sqrt{sZ_{DR}(u)}\,[s\rho_{hv}(u)V_V^{[1]}(u) + \sqrt{1 - s\rho_{hv}^2(u)}V_V^{[2]}(u)]e^{i\delta_{hv}(u)} \quad (3)$$

where the spectral variables $s\rho_{hv}$, $s\delta_{hv}$ and sZ_{DR} are presented in Figure 1, for each velocity bin, but also hold the prefix s in the notation to differentiate them from the commonly used integral polarimetric variables. $V_V^{[2]}$ is generated according to (2) with the same model spectrum S_{VV}, but with a second independent sequence of random numbers ($u^{[2]}$ and $\theta^{[2]}$). This process is repeated for each velocity bin for the total of the FFT spectral points within the Nyquist interval. The inverse Fourier transform of V_V and V_H represent simulated time series of complex signals for the V and H channels.

For the implementation of white noise, an approach similar to Eq. (2) is used:

$$N_V(u) = \sqrt{-\aleph_V \ln g^{[3]}}e^{i\theta^{[3]}} \quad \text{and} \quad N_H(u) = \sqrt{-\aleph_H \ln g^{[4]}}e^{i\theta^{[4]}} \quad (4)$$

where $\aleph_V$ and $\aleph_H$ are the noise level values for the V and H channel corresponding to the prescribed values of signal-to-noise ratios (SNR), and $u^{[3]}$, $\theta^{[3]}$, $u^{[4]}$, $\theta^{[4]}$ are again generated independently. The complex numbers that represent the simulation of the noisy I and Qs in the frequency domain for the V and H channels are calculated from:

$$S_V(u) = V_V(u) + N_V(u) \quad \text{and} \quad S_H(u) = V_H(u) + N_H(u) \quad (5)$$

To introduce the turbulent motions of drops in the simulations, the Doppler spectra must be convolved with a turbulence term S_{air} (6), that accounts for the turbulent motions within the atmosphere.

$$S_{air}(u) = \frac{1}{\sqrt{2\pi}\sigma_t} e^{-\frac{u^2}{2\sigma_t^2}} \quad (6)$$

$$S_{VV}^{turb} = \int_{-\infty}^{\infty} S_{VV}(u-\xi)S_{air}(\xi)d\xi \quad (7)$$

where ξ is the convolution variable and Sair accounts for the turbulent motions within the atmosphere. Atmospheric turbulence causes random fluctuations in the velocity of hydrometeors, thus broadening the Doppler spectrum. All droplets are here assumed to have no inertial effects and therefore acting like perfect tracers (Figure 2).

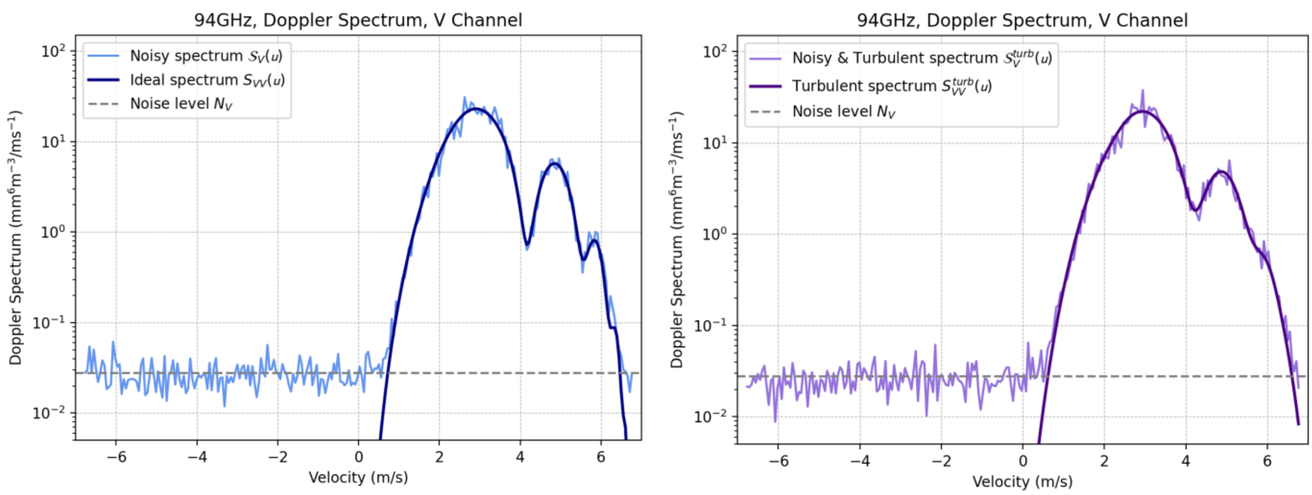

***Figure 2**: Left: Ideal Spectrum S_{VV} (eq. 1) – dark blue line, Noisy spectrum S_V (eq. 5) – light blue line, Right: Turbulent spectrum without noise effect (eq. 7) – dark purple line, Turbulent and noisy spectrum – light purple line. The grey dashed line, represents the noise level $\aleph_V$.*

Then the broadened sZ_{DR} can be computed as the ratio of $S^{turb}_{HH}(v)$ to $S^{turb}_{VV}(v)$ whereas the turbulent-broadened parameters ρ^{turb}_{HV} and δ^{turb}_{HV} are then calculated respectively as the amplitude and the phase of the variable:

$$\rho_{hv}(u)e^{i\delta_{hv}(u)} = \frac{<S_H(u)S_V^*(u)>}{\sqrt{<|S_H(u)|^2><|S_V(u)|^2>}} \quad (8)$$

3 Results

To assess the accuracy of the cloud radar simulation methods, we compare the measurements with the simulated data. This comparison aims to validate the performance of the simulations and identify any discrepancies that may arise from the model assumptions or parameter settings. The cloud radar measurements were obtained using an RPG Frequency Modulated Continuous Wave (FMCW) Dual Polarization W-band Cloud Doppler Radar, operating at 94 GHz.

One case study from 3 February 2021, is presented in Figure 3. The rainfall is moderate, with rain rate approximately between 6 and 7 mm/h. The spectrum is acquired at an altitude of 484 meters above ground level. At this altitude, there is significantly less turbulence relatively to lower levels, due to the influence of surface effects diminishes, leading to generally more stable and less turbulent atmospheric conditions.

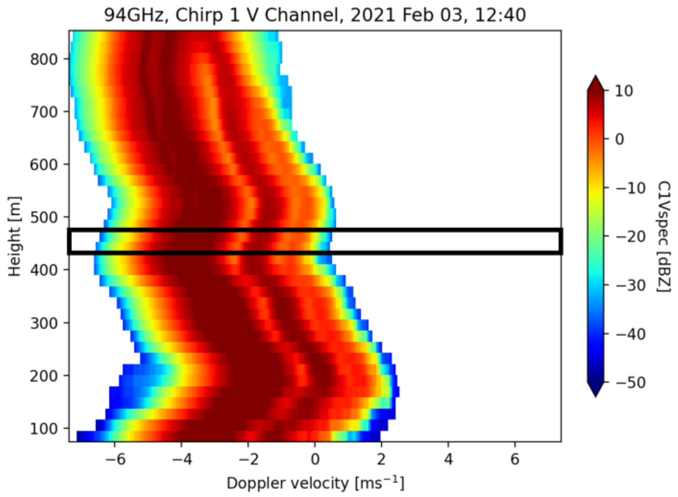

Figure 3*: 03 Feb 2021, 12:40z with vertical profiles for reflectivity. The level that is used for case studies are marked by the black rectangle.*

For the generation of the simulation of the spectral polarimetric variables, the optimal fit for the drop size distribution must be identified in order to apply equation 1. Therefore, the Least Squares Method was employed to minimize the sum of the squared differences between the measured and simulated spectra, ensuring that the best-fitting gamma DSD is selected. The best fit is presented in Figure 4.

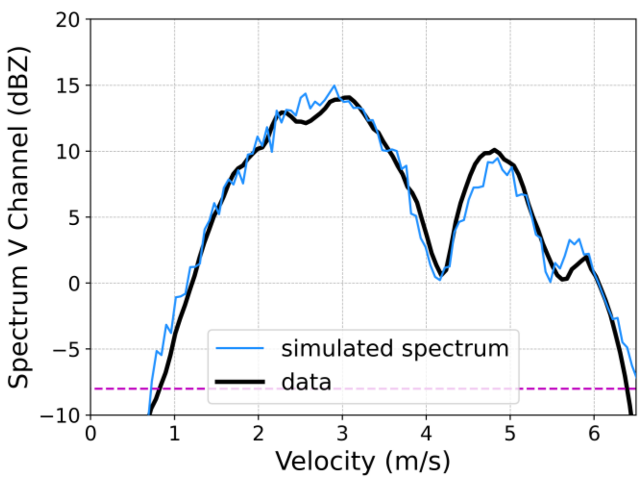

Figure 4*: Measured Doppler Spectrum (black line) and optimum-fitted Gamma DSD (blue line). The purple dashed line indicates the threshold for applying the Least Squares Method in order to find the optimum fit.*

The simulated variables as calculated from equation 8, are represented in figure 5. The black lines represent the measured sZ_{DR} (left) and $s\delta_{HV}$ (right), while the blue line is the simulation. The comparison between cloud radar simulations and measurements exhibits some correlation; however, there are notable discrepancies that indicate limitations in the current simulation model.

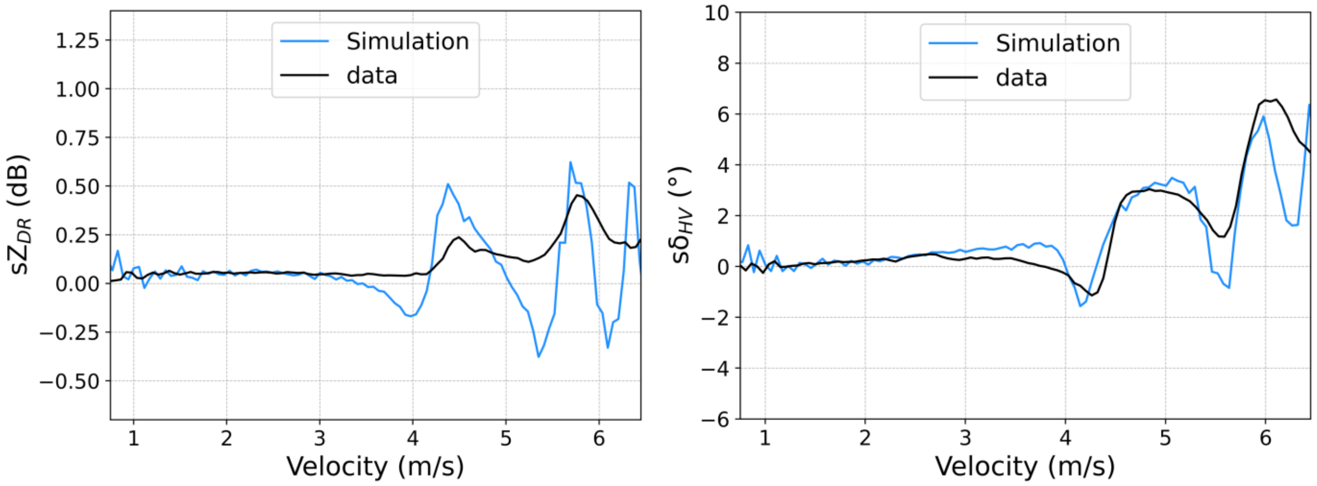

Figure 5*: Spectral polarimetric variables of case study. Left panel: Spectral Differential reflectivity* sZ_{DR}*, Right panel: Spectral Differential phase* $s\delta_{HV}$*. The black lines represent the measured data, the blue lines represent the simulations from the above-described method.*

We observe notable discrepancies in the simulations (blue lines) when compared to observational data. Specifically, the expected minima, as predicted by theoretical models, are not well-represented in the data, especially for Z_{DR}. For δ_{hv}, there is better agreement, though this deteriorates with larger drops. The simulations tend to produce more pronounced minima than those observed, suggesting potential issues in the microphysical assumptions, such as the parameterization of the drop shape. In our study, all drops are modelled as spheroids. In these cases, this T-matrix approach may fail at high frequencies, like 94 GHz.

4 Conclusions

This study compares simulated spectral polarimetric variables with real measurements from a 94 GHz cloud radar under moderate rain conditions. The results show that the simulations closely match observations within a specific spectrum range, particularly for Doppler velocities up to 5 m/s, where the polarimetric signal is minimal due to the predominantly spherical shape of raindrops. The simulations more accurately represent the maxima than the minima, specifically for the differential phase $s\delta_{HV}$. While the minima of the observed data for both sZ_{DR} and $s\delta_{HV}$ appear truncated, the simulated minima are significantly deeper. Despite these discrepancies, the overall trends in both simulations and measurements remain consistent. A potential explanation for these discrepancies may be found in the assumptions of the T-matrix approach, which models all drops as spheroidal or rotationally symmetric particles. Since raindrops oscillate and lack rotational symmetry, traditional methods like the T-matrix may produce inaccurate scattering parameters, particularly for resonant particles, where radar wavelengths are comparable to or smaller than raindrop size. More accurate methods should be explored. Future research should investigate whether more advanced scattering models can account for the observed variability, or alternatively, data from low turbulence conditions could be used to create look-up tables of polarimetric scattering properties based on incidence angle in a data-driven approach. This work contributes to the broader scientific effort to improve cloud radar simulations and advance the understanding of cloud processes and their impact on atmospheric dynamics.

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